

The Regulatory Assistance Project

# The E3-India Model

## Technical model manual, Volume 4: Model Specification



This volume (4) is part of a 9-volume series covering the E3-India model

June 2017

Cambridge Econometrics  
Cambridge, UK

info@camecon.com  
www.camecon.com

Cambridge Econometrics' mission is to provide rigorous, accessible and relevant independent economic analysis to support strategic planners and policy-makers in business and government, doing work that we are interested in and can be proud of.

Cambridge Econometrics Limited is owned by a charitable body,  
the Cambridge Trust for New Thinking in Economics.  
[www.neweconomicthinking.org](http://www.neweconomicthinking.org)

## Authorisation and Version History

---

Version	Date	Authorised for release by	Description
1.0	17/06/17	Hector Pollitt	First version, volume 4.

---

# Contents

---

	Page
1 Functions in E3-India	5
1.1 Introduction	5
1.2 Main macroeconomic identity relationships	5
1.3 Summary specification of equations	8
1.4 Aggregate energy demand	10
1.5 Disaggregate energy demand for coal, heavy fuel oil, gas and electricity	13
1.6 Household consumption	15
1.7 Industrial investment	18
1.8 The international trade equations	20
1.9 Export and import prices	23
1.10 Domestic industry prices	25
1.11 Industrial employment	27
1.12 Industrial average earnings	28
1.13 Labour participation rate	30

# 1 Functions in E3-India

## 1.1 Introduction

In common with other economic models, E3-India consists of a combination of accounting balances and behavioural relationships. This volume describes both types of equation in the following sections, starting with the main accounting relationships.

The modelling approach for the behavioural relationships is econometric, meaning that the basis for determining the relationships is the historical time-series data. The estimation methods used are described in detail in Volume 3, Section 3. The current version of E3-India includes 16 sets of econometric equations that cover energy consumption, several economic indicators, international trade and the labour market.

The following sections provide an overview of the equation specifications.

## 1.2 Main macroeconomic identity relationships

In this section, we present the main accounting equations. The model follows the structure of the National Accounts, with disaggregation by sector in each state.

### GDP, output and value added

The main measure of GDP is determined by demand-side factors in E3-India, following the definition shown below. This follows the standard accounting definition.

**Table 1.1: GDP identity**

$$RGDP = RSC + RSK + RSG + RSX - RSM + RSS$$

Definitions:

RGDP	is GDP, m Rs at 2010 prices
RSC	is total consumer expenditure, m Rs at 2010 prices
RSK	is total investment (GFCF), m Rs at 2010 prices
RSG	is total final government expenditure, m Rs at 2010 prices
RSX	is total exports, m Rs at 2010 prices
RSM	is total imports, m Rs at 2010 prices
RSS	is total inventories, m Rs at 2010 prices

**Table 1.2: Calculation of output**

QR	=	QRY + QRC + QRK + QRG + QRX - QRM + QRR
Definitions:		
QR		Is a vector of output (by product), m Rs at 2010 prices
QRY		is a vector of intermediate goods, m Rs at 2010 prices
QRC		is a vector of final consumer output goods, m Rs at 2010 prices
QRK		is a vector of final investment goods, m Rs at 2010 prices
QRG		is a vector of final government goods, m Rs at 2010 prices
QRX		is a vector of final exported goods, m Rs at 2010 prices
QRM		is a vector of final imported goods, m Rs at 2010 prices
QRR		is a residual value to balance accounts, m Rs at 2010 prices

**Output** While GDP provides a measure of net production at the whole-economy level, at the sectoral level we have (gross) output and gross value added. Output is equivalent to turnover in that it includes intermediate inputs to production, while value added does not include purchases from other sectors.

The measure of output is determined from the demand side, in a similar way to GDP but also including the intermediate demands, as shown below. Each variable in the box is defined by both state and sector.

A fundamental part of the national accounting structure is that supply and demand must match. In the demand-driven structure of E3-India this is imposed by ensuring that production matches the level of the goods demanded (if there are supply constraints that prevent this from happening then demand must be adjusted separately).

The basic relationship is presented below.

**Table 1.3: Balancing supply and demand**

YR	=	QR
Definitions:		
YR		is a vector of output (by industry), m Rs at 2010 prices
QR		is a vector of output (by product), m Rs at 2010 prices

**Value added** Value added is defined as the difference between output and material input costs. Value added itself is the sum of wages, company profits and production taxes.

GDP and value added are among the most important model results but there are other identity relationships that play an important role in determining these results. The key ones are presented in the following paragraphs, starting with the measures of consumer prices and inflation.

**Table 1.4: Calculating GVA**

$$YRF = YR - YRQ - YRT$$

Definitions:

YRF is a vector of value added, m Rs at 2010 prices

YR is a vector of output (by industry), m Rs at 2010 prices

YRQ is a vector of intermediate demands by industry, m Rs at 2010 prices

YRT is a vector of taxes on products, m Rs at 2010 prices

## Consumer prices

Consumer prices are determined by converting industry prices to the relevant consumer products. For example, the prices of cars are determined by the output prices of the car industry, plus the contribution from transport and retail costs, plus the taxes on purchases of new cars.

The general structure of the relationships is shown below.

**Table 1.5: Consumer prices**

$$PCR = (BQRC * PQRD * CR) * ((1+CRTR) / CR)$$

Definitions:

PCR is a vector of consumer prices, by product, m Rs at 2010 prices

BQRC is a matrix that converts industry production to consumer products

PQRD is a vector of prices of industry sales to the domestic market, m Rs at 2010 prices

CR is a vector of consumer products, m Rs at 2010 prices

CRTR is a vector of indirect tax rates on consumer products

## The consumer price index

The aggregate consumer price index is obtained by taking the sum across all consumer products. Inflation is the annual change in the consumer price index.

**Table 1.6: The consumer price index**

$$PRSC = \text{sum}(PCR * CR) / RSC$$

Definitions:

PRSC is the aggregate consumer price index, 2010 = 1.0

PCR is a vector of consumer products' prices, 2010 = 1.0

CR is a vector of expenditure on consumer products, m Rs at 2010 prices

RSC is the sum of expenditure on consumer products, m Rs at 2010 prices

## Household real incomes

Real incomes are the main driver of consumption, which is the largest component of GDP. The level of real incomes is therefore a key model result. The variable is determined by summing wage and non-wage income in nominal terms, and converting to real terms.

Non-wage income includes rents from property and other financial and non-financial assets, plus remittances. It is very difficult to model and is held as a

fixed differential to wage income (i.e. if wage income increases by 2% then it is assumed that non-wage income increases by 2% as well).

**Table 1.7: Calculating real incomes**

RRPD	=	(sum (YRW * YRE) + RRI) / PRSC
Definitions:		
RRPD	is a measure of real household income, m Rs at 2010 prices	
YRW	is the average annual wage in each sector, th Rs	
YRE	is the employment level in each sector, th people	
RRI	is a measure of non-wage ('residual') income, m Rs	
PRSC	is the aggregate consumer price index, 2010 = 1.0	

The remainder of this volume focuses on the econometric equations in the model.

### 1.3 Summary specification of equations

Table 1.8 provides a list of the estimated equations. Table 1.9 summarises the variables that are used and units of measurement. A full list of model variables is available on request.

#### Dummy variables

The use of dummy variables in E3-India modelling is restricted by the limited degrees of freedom offered by the time-series data but there is an important case where a dummy variable is added to all the equation sets

The financial crisis in 2009 provoked many non-linear reactions. To reduce bias in our parameter estimates, a dummy variable for 2009 (zero before 2009, one from 2009 onwards) is included in all the equation sets.

To avoid excessive repetition, the dummy variable is not included in the formal definitions provided in the rest of this volume, but it is an important part of the model estimation and solution.

**Table 1.8: The econometric functions**

	Short Name	Description
1	BFR0	Aggregate Energy Demand
2	BFRC	Coal Demand
3	BFRO	Heavy Oil Demand
4	BFRG	Natural Gas Demand
5	BFRE	Electricity Demand
6	BRSC	Aggregate Consumption
7	BCR	Disaggregate Consumption
8	BKR	Industrial Investment
9	BQRM	Imports
10	BQRX	Exports
11	BYRE	Industrial Employment
12	BPYH	Industrial Prices
13	BPQX	Export Prices
14	BPQM	Import Prices
15	BYRW	Industrial Average Earnings
16	BLRP	Labour Participation Rate

**Table 1.9: Summary of the econometric equations**

	Endo var	V1	V2	V3	V4	Units
1	FR0	FRY	PREN	FRKE		th toe
2-5	FR(fuel)	FR0	PFRF	FRKE		th toe
6	RSC	RRPD	RRLR	RUNR	PRSC	m Rs 2010 prices
7	CR	RRPD	PRCR	RRLR	PRSC	consumption ratio
8	KR	YR	PKR/PYR	RRLR		m Rs 2010 prices
9	QM0	QRDI	PYH/PQM	YRKE		m Rs 2010 prices
10	QRX	QRDW	PQX	YRKE		m Rs 2010 prices
11	YRE	YR	LYLC	PQMA		thousands
12	PYH	YRUC	PQM	YRKE		index 20010=1.0
13	PQRX	PQWE	EX	YRULT		index 20010=1.0
14	PQM	PQWE	EX	YRUL		index 2010=1.0
15	YRW	LYWE	YRWE	LYRP	RUNR	th Rs per year
16	LRP	RSQ	RWS/REMP	RUNR		rate [0,1]

The names of variables and parameter sets closely follow the conventions for Fortran names, i.e. they are groups of capital letters and numbers beginning with a letter.

**Table 1.10: Understanding the function tables**

+ - * and /	denote addition, subtraction, multiplication and division of scalars and of individual elements of vectors and matrices.
()	are grouping brackets.
[]	enclose comments.
(.)	as a postscript on a name indicates that it is a vector with the dot denoting all the elements.
(..)	as a postscript on a name indicates that it is a matrix.
(^)	denotes that the vector is converted to a diagonal matrix.
(.,.)'	denotes that the matrix is transposed.
(-1), (-2) etc.	as applied to a variable or a group of variables as a postscript, denote a one, two etc. period lag. (This may also be done by adding L1, L2 etc. or 1, 2 etc. to the FORTRAN name.)
LN(V)	natural logarithm of variable V.
DLN(V)	change in LN(V).
MATP(M1(..),M2(..))	matrix multiplication of variable matrices M1 and M2.

Nearly all the variables and parameters are defined by state. In order to reduce the complexity of the notation, this regional dimension is omitted in the tables below. Therefore, all variables and parameters should be assumed to vary over the states of India unless otherwise stated.

#### 1.4 Aggregate energy demand

The original equation is based on work by Barker, Ekins and Johnstone (1995) and Hunt and Manning (1989). The work by Serletis (1992), and Bentzen and Engsted (1993) has also helped in forming the specification for the cointegrating equation. The version of the equation in the E3ME global model has been adapted for E3-India.

#### Overall structure

Since there are substitutable inputs between fuels, the total energy demand in relation to the output of the energy-using industries is likely to be more stable than the individual components. Even so, total energy demand is also subject to considerable variation, which reflects both technical progress in conservation, and changes in the cost of energy relative to other inputs. The aggregate energy equation considers the total energy used (summation of five carriers) in thousand tonnes of oil equivalent (th toe) by each energy user. The demand for energy is dependent on the economic 'activity' for that user (converted from the 20 economic sectors). This is chosen as gross economic output for most sectors, but household energy demand is a function of total consumers' expenditure. A restriction is imposed so that higher activity does not result in lower energy use (all other factors being equal).

The average price used in the equations weights the prices of individual energy carriers by their share in consumption by each user. Due to data limitations, the current energy demand equations do not allow for asymmetrical effects (i.e. rising energy prices leading to reductions in fuel demand, but falling prices not leading to an increase). Such asymmetrical price effects in aggregate energy demand equations have been the subject of other research (Gately, 1993; Walker and Wirl, 1993; Grubb, 1995, 2014). The idea is that because energy is

used via capital stock with a long lifetime, and since technical change is progressive and is not generally reversed, when energy prices rise and energy savings are introduced, then when energy prices fall again, these savings are not reversed i.e. energy demand responds to rises in real prices, but not falls. This will be revisited in future.

**Price elasticities** As described in Volume 5, Section 2.2, the long-run price elasticities are taken from the literature rather than estimated using the time-series data. The long-run price elasticity for transport is imposed at -0.45 for all states, following the research on long-run road transport demand (Franzen and Sterner, 1995) and (Johansson and Schipper, 1997, p. 289). CE's internal research, using cross-sectional analysis of the E3ME data set has confirmed this result. Elasticities for other sectors are around -0.2.

**Technology and capital stock** The measures of research and development expenditure capture the effect of new ways of decreasing energy demand (energy saving technical progress) and the elimination of inefficient technologies, such as energy saving techniques replacing the old inefficient use of energy. The variable FRKE is determined by converting the economic estimates for the technological progress indicators into the energy using categories.

**The power sector** The power generation sector is solved using the bottom-up FTT model (see Volume 5, Section 3) rather than the estimated equations. The top-down approach offered by the econometric equations is not appropriate for this sector because:

- there is a small number of large plants, meaning estimated parameters give a poor performance
- the econometric approach is not well suited to the development of new renewable technologies

Table 1.11: Aggregate energy demand equations

<i>Co-integrating long-term equation:</i>	
LN(FR0(.))	[total energy used by energy user]
= BFR0(.,7)	
+ BFR0(.,8) * LN(FRY(.))	[activity measure]
+ BFR0(.,9) * LN(PREN(.))	[average price ratio]
+ BFR0(.,10) * LN(FRKE(.))	[technology measure]
+ ECM	[error]
<i>Dynamic equation:</i>	
DLN(FR0(.))	[total energy used by energy user]
= BFR0(.,1)	
+ BFR0(.,2) * DLN(FRY(.))	[activity measure]
+ BFR0(.,3) * DLN(PREN(.))	[average price ratio]
+ BFR0(.,4) * DLN(FRKE(.))	[technology measure]
+ BFR0(.,5) * DLN(FR0(-1))	[lagged change in energy use]
+ BFR0(.,6) * ECM(-1)	[lagged error correction]
<i>Identity:</i>	
PREN = PFR0(.) / PRYR	[relative price ratio]
<i>Restrictions:</i>	
BFR0(.,3 .,4 .,9 .,10) <= 0	[‘right sign’]
BFR0(.,2 .,8) >= 0	[‘right sign’]
0 > BFR0(.,6) > -1	[‘right sign’]
<i>Definitions:</i>	
BFR0	is a matrix of parameters
FR0	is a matrix of total energy used by energy user, th toe
PFR0	is a matrix of average energy prices by energy user, Rs/toe
PRYR	is a matrix of average producer prices in the economy as a whole, 2010 = 1.0
FRY	is a matrix of activity by energy user, m Rs at 2010 prices
FRKE	is a matrix of technological progress by industry, converted to energy users

## 1.5 Disaggregate energy demand for coal, heavy fuel oil, gas and electricity

The specification is shown in Table 1.12.

The equations for disaggregated energy demand have been specified for four energy carriers<sup>1</sup>: coal, oils, gas and electricity. There is no reliable price data for biomass to form econometric relationships. Biomass in E3-India is therefore treated as a residual fuel and is set to move in line with total energy demand.

The four carriers that are modelled have the characteristic that in some industries they are highly substitutable inputs to the process of heat generation. The specification of the equations follows similar lines to the aggregate energy demand equations (see previous section). The equations contain the same technology variable, with the same restrictions imposed. Instead of using a measure of economic activity, total energy consumption by the sector is used.

The price term is a ratio of the price for the particular energy carrier in question to that of the aggregate energy price. The relative fuel prices have changed dramatically over the period of historical data, particularly towards the start and end of the time series.

Again, the power generation sector is solved using the FTT submodel, and does not use the estimated equation.

---

<sup>1</sup> These are also referred to as 'fuels' for brevity.

**Table 1.12: Disaggregate energy demand equations**

<i>Equations used for F = Coal (C), Heavy Fuel Oil (O), Natural Gas (G) and Electricity (E)</i>		
<i>Co-integrating long-term equation:</i>		
$\text{LN}(\text{FRF}(.))$		[fuel used by energy user]
=	$\text{BFRF}(. , 7)$	
+	$\text{BFRF}(. , 8) * \text{LN}(\text{FR0}(.))$	[total energy used by energy user]
+	$\text{BFRF}(. , 9) * \text{LN}(\text{PFRP}(.))$	[price ratio]
+	$\text{BFRF}(. , 10) * \text{LN}(\text{FRKE} (.))$	[technology index]
+	$\text{ECM}$	[error]
<i>Dynamic equation:</i>		
$\text{DLN}(\text{FRF}(.))$		[fuel used by energy user]
=	$\text{BFRF}(. , 1)$	
+	$\text{BFRF}(. , 2) * \text{DLN}(\text{FR0}(.))$	[total energy used by energy user]
+	$\text{BFRF}(. , 3) * \text{DLN}(\text{PFRP}(.))$	[price ratio]
+	$\text{BFRF}(. , 4) * \text{DLN}(\text{FRKE} (.))$	[technology index]
+	$\text{BFRF}(. , 5) * \text{DLN}(\text{FRF}(-1))$	[lagged change in energy use]
+	$\text{BFRF}(. , 6) * \text{ECM}(-1)$	[lagged error correction]
<i>Identity:</i>		
$\text{PFRP}$	= $\text{PFRF}(.)/\text{PFR0}(.)$	[price ratio]
<i>Restrictions:</i>		
$\text{BFRF}(. , 3 \dots 4 \dots 9 \dots 10) \leq 0$		['right sign']
$\text{BFRF}(. , 2 \dots 8) \geq 0$		['right sign']
$0 > \text{BFRF}(. , 6) > -1$		['right sign']
<i>Definitions:</i>		
$\text{BFRF}$	is a matrix of parameters	
$\text{FRF}$	is a matrix of fuel used by energy user, th toe	
$\text{FR0}$	is a matrix of total energy used by energy user, th toe	
$\text{PFRF}$	is a matrix of prices for energy carrier F, by energy user, Rs/toe	
$\text{PFR0}$	is a matrix of average energy prices by energy user, Rs/toe	
$\text{FRKE}$	is a matrix of technological progress by industry, converted to energy users	

## Aggregate household consumption

### 1.6 Household consumption

The model equations for household consumption are split into two separate sets. The first set estimates total consumption volumes, while the second set allocates this consumption according to the available budget.

The equation specification is given in Table 1.13. It should be noted that the dependent variable and term for income are converted into per capita measures, although this is excluded from the table for conciseness. As consumption accounts for around 40-50% of final demand the equation is very important within the model structure as a whole.

Most studies have followed those of Hendry et al (1978) which have examined the dynamic links between consumption, income and wealth in an error correction model. In more recent studies, attention has focused more upon the role of wealth (housing wealth in particular) and financial liberalisation (Barrell and Davis, 2007; Carruth and Kerdrain, 2011); current data in India do not allow for this type of assessment but it could be added to the model in future.

The specification of the equation is similar to that used in the previous HERMES and E3ME models, which generalise the permanent income and the lifecycle theories in an error correction model. Indeed, the long-run elasticity of consumption in relation to income has been set equal to one to ensure the lifecycle theory is fulfilled (wealth effects are missing from the equations in E3-India due to data constraints).

These equations relate total consumption to regional personal disposable income, unemployment rates, inflation and interest rates. The unemployment rate is used as a proxy for the degree of uncertainty in the economy and has been found to have significant effects on short-term consumption levels. As unemployment data in India can be unreliable, inactive population (i.e. working age population minus those employed) is used as a proxy.

**Table 1.13: Aggregate consumption equations**

<i>Co-integrating long-term equation:</i>		
LN(RSC)		[real consumers' expenditure]
=	BRSC(8)	
+	BRSC(9) * LN(RRPD)	[real gross disposable income]
+	BRSC(10) * LN(RRLR)	[real rate of interest]
+	ECM	[error]
<i>Dynamic equation:</i>		
DLN(RSC)		[real consumers' expenditure]
=	BRSC(1)	
+	BRSC(2) * DLN(RRPD)	[real gross disposable income]
+	BRSC(3) * DLN(RRLR)	[real rate of interest]
+	BRSC(4) * LN(RUNR)	[unemployment rate]
+	BRSC(5) * DLN(RPSC)	[consumer price inflation]
+	BRSC(6) * DLN(RSC(-1))	[lagged change in consumers' expenditure]
+	BRSC(7) * ECM(-1)	[lagged error correction]
<i>Identities:</i>		
RRLR	=	1 + (RLR-DLN(PRSC))/100 [real rate of interest]
RRPD	=	(RGDI / PRSC) [real gross disposable income]
<i>Restrictions:</i>		
BRSC(9) = 1		['life cycle hypothesis']
BRSC(2) >= 0		['right sign']
BRSC(3, 4, 5, 10) <= 0		['right sign']
0 > BRSC(7) > -1		['right sign']
<i>Definitions</i>		
BRSC	is a matrix of parameters	
RSC	is a vector of total consumers' expenditure, m Rs at 2010 prices	
RGDI	is a matrix of gross disposable income, m Rs at current prices	
RRLR	is a matrix of long-run nominal interest rates	
RUNR	is a vector of unemployment rates, measured as a percentage of the labour force	
PRSC	is a vector of consumer price deflator, 2010=1.0	
RPSC	is a vector of consumer price inflation, in percentage terms	

## Disaggregate consumption

The specification is shown in Table 1.14

Both the long-term and dynamic equations have a similar specification to the aggregation consumption equations, but include the relative prices of each consumption category.

**Table 1.14: Disaggregate consumption equations**

<i>Co-integrating long-term equation:</i>		
LN(SHAR(.))		[consumers' budget share, logistic form]
=	BCR(.,8)	
+	BCR(.,9) * LN(RRPD)	[real gross disposable income]
+	BCR(.,10) * LN(PRCR(.))	[relative price of consumption]
+	BCR(.,11) * LN(RRLR)	[real rate of interest]
+	BCR(.,12) * LN(PRSC)	[consumer price deflator]
+	ECM	[error]
<i>Dynamic equation:</i>		
DLN(SHAR(.))		[consumers' budget share, logistic form]
=	BCR(.,1)	
+	BCR(.,2) * DLN(RRPD)	[real gross disposable income]
+	BCR(.,3) * DLN(PRCR(.))	[relative price of consumption]
+	BCR(.,4) * DLN(RRLR)	[real rate of interest]
+	BCR(.,5) * DLN(PRSC)	[consumer price deflator]
+	BCR(.,6) * DLN(SHAR)(-1)	[lagged change in consumers' budget share]
+	BCR(.,7) * ECM(-1)	[lagged error correction]
<i>Identities:</i>		
SHAR	= (VCR(.)/VCRT) / (1-(VCR(.)/VCRT))	[consumers' budget share, logistic form]
RRPD	= (RGDI/PRSC)/RPOP	[real gross disposable income]
PRCR	= VCR(.)/CR(.)/PRSC	[real price of consumption]
RRLR	= 1+(RLR-DLN(PRSC))/100	[real rate of interest]
<i>Restriction:</i>		
0 > BCR(.,7) > -1		['right sign']
<i>Definitions:</i>		
BCR	is a matrix of parameters	
CR	is a matrix of consumers' expenditure by commodity, m Rs at 2010 prices	
VCR	is a matrix of consumers' expenditure by commodity, m Rs at current prices	
VCRT	is a vector of total consumers' expenditure, m Rs at current prices	
RGDI	is a matrix of gross disposable income, in m Rs at current prices	
RLR	is a matrix of long-run nominal interest rates	
PRSC	is a vector of total consumer price deflator, in percentage terms	
RPSC	is a vector of consumer price inflation, in percentage terms	

## 1.7 Industrial investment

### Industrial investment

Investment (see Table 1.15) is a very important and very volatile component of final demand, so its treatment in the model is of central importance to model simulation and forecasting performance. Ideally, the treatment of investment in a sectoral model such as E3-India should disaggregate by asset (e.g. vehicles, plant and machinery, and buildings) as well as by investing industry, but this has not proved possible due to data limitations.

The specification of the investment equations in E3-India has built upon earlier work in the E3ME model and published in Barker and Peterson (1987). The theory behind the choice of variables that explain the long-run path of investment is a mix between the neoclassical tradition, whereby factor demands are explained solely in terms of other factor prices, and the accelerator model, which recognises the importance of output as a determining influence. For the dynamic representation, the real rate of interest is also added.

E3-India is bound by the investment-savings national accounts identity but the representation of capital markets in E3-India does not assume a fixed stock of money, as is typically the case in CGE models. Endogenous money is an important feature of the model, with banks creating money whenever they see a profitable lending opportunity (Pollitt and Mercure, 2017).

Table 1.15: Investment equations

<i>Co-integrating long-term equation:</i>		
LN(KR(.))		[investment]
=	BKR(.,7)	
+	BKR(.,8) * LN(YR(.))	[real output]
+	BKR(.,9) * LN(PKR.)/PYR(.))	[relative price of investment]
+	ECM	[error]
<i>Dynamic equation:</i>		
DLN(KR(.))		[change in investment]
=	BKR(.,1)	
+	BKR(.,2) * DLN(YR(.))	[real output]
+	BKR(.,3) * DLN(PKR.)/PYR(.))	[relative price of investment]
+	BKR(.,4) * LN(RRLR)	[real rate of interest]
+	BKR(.,5) * DLN(KR)(-1)	[lagged change in investment]
+	BKR(.,6) * ECM(-1)	[lagged error correction]
<i>Identities:</i>		
RRLR	= 1 + (RLR – DLN(PRSC)) / 100	[real rate of interest]
<i>Restrictions:</i>		
BKR(.,2 .,8) >= 0		['right sign']
BKR(.,3 .,4 .,9) <= 0		['right sign']
0 > BKR(.,6) > -1		['right sign']
<i>Definitions:</i>		
BKR	is a matrix of parameters	
KR	is a matrix of investment expenditure by industry, m Rs at 2010 prices	
YR	is a matrix of gross industry output by industry, m Rs at 2010 prices	
PKR	is a matrix of industry investment price by industry, 2010=1.0	
PRSC	is a vector of consumer price deflator, 2010=1.0	
RLR	is a vector of long-run nominal interest rates	
PYR	is a matrix of industry output price by industry, 2010=1.0, Rs	

## 1.8 The international trade equations

Trade is an important feature in the E3-India model for two main reasons. Firstly, globalisation has meant that international trade has accounted for an increasing share of total production (expected to increase further in the future, even with slower future trade growth). Secondly, exports and imports represent the linkage between India and the rest of the world, so effects moving from one India to another country, or from another country to India, are transmitted via this area of the model.

In a sub-national model, trade represents a major issue in assessing regional economic impacts. Demand in each state can be met either by production within that state, production in another state, or production in another country. With no available data on trade between the states, it is necessary to impose assumptions on the rates of production in the states with relation to developments in neighbouring states.

The approach can be summarised as:

- International exports are estimated at state level, based on the production prices within each state.
- International imports are estimated at national level and applied to the states, based on estimates of current and baseline future state-level imports.
- Trade between states is estimated using production shares that could be varied in response to changes in prices within each state.

### International Imports

In the import equations, activity is modelled by sales to the domestic market, the relative price of sales to the domestic market and the technical progress variable.

### International export volumes

In the E3-India model, exports are explained as a function of the demand of the rest of world for Indian production, export prices and the technology variable. The technology variable is included to allow for the effects of innovations on trade performance.

The formal specification of the import and export equations is shown in Table 1.16 and Table 1.17.

**Table 1.16: International export volume equations**

<i>Co-integrating long-term equation:</i>	
$LN(QRX(.))$	[export volume]
= BQRX(.,7)	
+ BQRX(.,8) * LN(QRDW(.))	[rest of the world demand]
+ BQRX(.,9) * LN(PQRX(.))	[exports price]
+ BQRX(.,10) * LN(YRKE(.))	[technological progress]
+ ECM	[error]
<i>Dynamic equation:</i>	
$DLN(QRX(.))$	[change in internal export volume]
= BQRX(.,1)	
+ BQRX(.,2) * DLN(QRDW (.))	[rest of the world demand]
+ BQRX(.,3) * DLN(PQRX(.))	[exports price]
+ BQRX(.,4) * DLN(YRKE(.))	[technological progress]
+ BQRX(.,5) * DLN(QRX)(-1)	[lagged change in export volume]
+ BQRX(.,6) * ECM(-1)	[lagged error correction]
<i>Restrictions:</i>	
$BQRX(.,2 \text{ } .,4 \text{ } .,8 \text{ } .,10) \geq 0$	['right sign']
$BQRX(.,3 \text{ } .,9) \leq 0$	['right sign']
$0 > BQRX(.,6) > -1$	['right sign']
<i>Definitions:</i>	
BQRX	is a matrix of parameters
PQRX	is a matrix of export prices by industry, 2010=1.0
QRDW	is a matrix of production in the rest of the world, m Rs at 2010 prices
QRX	is a matrix of exports by industry, m Rs at 2010 prices
YRKE	is a matrix of technological progress by industry

Table 1.17: International import volume equations

<i>Co-integrating long-term equation:</i>		
LN(QM0(.))		[import volume]
=	BQRM(.,7)	
+	BQRM(.,8) * LN(QRDI(.))	[home sales]
+	BQRM(.,9) * LN(PYH(.)/PQRM(.))	[relative price]
+	BQRM(.,10) * LN(YRKE(.))	[technological progress]
+	ECM	[error]
<i>Dynamic equation:</i>		
DLN(QM0(.))		[change in internal import volume]
=	BQRM(.,1)	
+	BQRM(.,2) * DLN(QRDI(.))	[home sales]
+	BQRM(.,3) * DLN(PYH(.)/PQRM(.))	[relative price]
+	BQRM(.,4) * DLN(YRKE(.))	[technological progress]
+	BQRM(.,5) * DLN(QRM)(-1)	[lagged change in import volume]
+	BQRM(.,6) * ECM(-1)	[lagged error correction]
<i>Identity:</i>		
QRDI	= QR(.) + QRM(.)	[home sales]
PYH	= (VQR(.) - VQRX(.)) / (QR(.) - QRX(.))	[price home sales by home producers]
<i>Restrictions:</i>		
BQRM(.,2 .,3 .,8 .,9) >= 0		['right sign']
BQRM(.,4 .,10) <= 0		['right sign']
0 > BQRM(.,6) > -1		['right sign']
<i>Definitions:</i>		
BQRM	is a matrix of parameters	
PQM	is a vector of import prices by industry, 2010=1.0	
QR	is a vector of gross output by industry (here, aggregated over states), m Rs at 2010 prices	
QM0	is a vector of imports to India by industry, m Rs at 2010 prices	
QRX	is a matrix of exports by industry (here, aggregated over states), m Rs at 2010 prices	
YRKE	is a matrix of technological progress by industry (here, aggregated over states)	
V-	indicates a current price version of the variable	

## 1.9 Export and import prices

The basic model of trade prices used in E3-India assumes that each sector operates in oligopolistic markets and is small in relation to the total Indian and global markets. Certain commodities, e.g. crude mineral oil, have prices treated exogenously, but the majority are treated in the following manner. Following from the assumption on market structure, prices are set by producers as mark-ups on costs, i.e. unit costs of production. Aside from this, the same variables are used for both import and export prices, within a general log-log functional form.

Alongside the unit cost variable, there is a price term included in each regression to deal with developments outside India.

Restrictions are imposed to force price homogeneity on the long-term equations, again in much the same manner as for the trade volume equations.

**Table 1.18: Export price equations**

<i>Co-integrating long-term equation:</i>	
$\text{LN}(\text{PQRX}(.))$	[export price]
= $\text{BPQX}(.,6)$	
+ $\text{BPQX}(.,7) * \text{LN}(\text{PQWE}(.)*\text{EX})$	[world commodity prices]
+ $\text{BPQX}(.,8) * \text{LN}(\text{YRULT}(.))$	[unit labour and tax costs]
+ $\text{ECM}$	[error]
<i>Dynamic equation:</i>	
$\text{DLN}(\text{PQRX}(.))$	[change in export prices]
= $\text{BPQX}(.,1)$	
+ $\text{BPQX}(.,2) * \text{DLN}(\text{PQWE}(.)*\text{EX})$	[world commodity prices]
+ $\text{BPQX}(.,3) * \text{DLN}(\text{YRULT}(.))$	[unit labour and tax costs]
+ $\text{BPQX}(.,4) * \text{DLN}(\text{PQRX})(-1)$	[lagged change in export prices]
+ $\text{BPQX}(.,5) * \text{ECM}(-1)$	[lagged error correction]
<i>Identities:</i>	
$\text{PQWE} = \text{QMC}(.)*\text{PM}$	[world commodity price index]
$\text{YRULT} = (\text{YRLC}(.)+\text{YRT}(.))/\text{QR}(.)$	[unit labour and tax costs]
<i>Restrictions:</i>	
$\text{BPQM}(.,7) = 1 - \text{BPQM}(.,9)$	[price homogeneity]
$\text{BPQX}(.,2 \text{ } .,3 \text{ } .,7 \text{ } .,8) \geq 0$	['right sign']
$0 > \text{BPQX}(.,5) > -1$	['right sign']
<i>Definitions:</i>	
$\text{BPQX}$	is a matrix of parameters
$\text{EX}$	is a vector of exchange rates, Rs per \$, 2010=1.0
$\text{QMC}$	is a converter matrix between industries and the world commodity classification
$\text{PM}$	is a vector of commodity prices (in Rs) for 7 commodities, 2010=1.0
$\text{YRLC}$	is a matrix of employer labour costs by industry, Rs at current prices
$\text{YRT}$	is a matrix of tax costs, by industry, m Rs at current prices
$\text{QR}$	is a matrix of gross output by industry, m Rs at 2010 prices

Table 1.19: Import price equations

<i>Co-integrating long-term equation:</i>	
$\text{LN}(\text{PQM}(.))$	[import price]
= $\text{BPQM}(.,7)$	
+ $\text{BPQM}(.,8) * \text{LN}(\text{PQWE}(.)*\text{EX})$	[world commodity prices]
+ $\text{BPQM}(.,9) * \text{LN}(\text{YRULT}(.))$	[unit labour and tax costs]
+ $\text{ECM}$	[error]
<i>Dynamic equation:</i>	
$\text{DLN}(\text{PQM}(.))$	[change in export prices]
= $\text{BPQM}(.,1)$	
+ $\text{BPQM}(.,2) * \text{DLN}(\text{PQWE}(.)*\text{EX})$	[world commodity prices]
+ $\text{BPQM}(.,4) * \text{DLN}(\text{YRULT}(.))$	[unit labour and tax costs]
+ $\text{BPQM}(.,5) * \text{DLN}(\text{PQRX})(-1)$	[lagged change in export prices]
+ $\text{BPQM}(.,6) * \text{ECM}(-1)$	[lagged error correction]
<i>Identities:</i>	
$\text{PQWE} = \text{QMC}(.)*\text{PM}$	[world commodity price index]
$\text{YRULT} = (\text{YRLC}(.)+\text{YRT}(.)) / \text{QR}(.)$	[unit labour and tax costs]
<i>Restrictions:</i>	
$\text{BPQM}(.,8) = 1 - \text{BPQM}(.,9)$	[price homogeneity]
$\text{BPQM}(.,2, .4, .8, .9) \geq 0$	['right sign']
$0 > \text{BPQM}(.,6) > -1$	['right sign']
<i>Definitions:</i>	
$\text{BPQM}$	is a matrix of parameters
$\text{PQM}$	is a vector of imports to India, by industry, m Rs at 2005 prices
$\text{EX}$	is a vector of exchange rates, Rs per \$, 2005=1.0
$\text{QMC}$	is a converter matrix between industry and world commodity classifications
$\text{PM}$	is a vector of commodity prices (in Rs) for 7 commodities, 2005=1.0
$\text{YRLC}$	is a matrix of employer labour costs by industry (here, aggregated over states), Rs at current prices
$\text{YRT}$	is a matrix of tax costs, by industry (here, aggregated over states), m Rs at current prices
$\text{QR}$	is a matrix of gross output by industry (here, aggregated over states), m Rs at 2005 prices
$\text{PQRX}$	is a matrix of export prices for 56 industries (here, aggregated over states), 2005=1.0, local currency

## 1.10 Domestic industry prices

The following model of industry price formation was developed from Lee (1988), having previously been derived from Layard et al (1991). The original empirical results were presented in E3ME working paper no. 43 (Barker and Gardiner, 1994).

The basis for price setting is a measure of unit costs, which is formed by summing labour material and taxation costs, and dividing this by sectoral output. Each industry is assumed to produce a homogenous product but does not necessarily operate in a fully competitive market place. The degree to which cost increases are passed on in final product prices is determined by the level of competition in the sector.

Although import prices are included in unit costs, depending on the import content of production, import prices are added separately in the equation to allow for the effects of international competition on domestic price formation. In the long-term relationship, homogeneity is imposed between higher domestic and import cost effects, so that their combined impact is unitary. The equations also include the technology indices, as a higher quality product may command a higher price.

Some sectors have a specific treatment of price and do not use the estimated equations, instead using a simpler relationship:

- The electricity sector – based on long-run ‘levelised’ costs, or could be modelled as part of a regulated system.
- Government sectors – these are assumed to move in line with aggregate consumer price inflation.
- Regulated sectors – these are also assumed to move in line with aggregate consumer price inflation.

Table 1.20: The Domestic Industry Prices Equations

<i>Co-integrating long-term equation:</i>		
LN(PYH(.))		[price of home sales by home producers]
=	BPYH(.,7)	
+	BPYH(.,8) * LN(YRUC(.))	[unit costs]
+	BPYH(.,9) * LN(PQRM(.))	[import price]
+	BPYH(.,10) * LN(YRKE(.))	[technological progress]
+	ECM	[error]
<i>Dynamic equation:</i>		
DLN(PYH(.))		[change in price of home sales by home producers]
=	BPYH(.,1)	
+	BPYH(.,2) * DLN(YRUC(.))	[unit costs]
+	BPYH(.,3) * DLN(PQRM(.))	[import price]
+	BPYH(.,4) * DLN(YRKE(.))	[technological progress]
+	BPYH(.,5) * DLN(PYH)(-1)	[lagged change in price]
+	BPYH(.,6) * ECM(-1)	[lagged error correction]
<i>Identities:</i>		
PYH	= (VQR(.) - VQRX(.)) / (QR(.) - QRX(.))	[price of home sales by home producers]
YRUC	= YRUM(.,) + YRUL(.) + YRUT(.)	[unit costs]
YRUL	= YRLC(.) / YR(.)	[unit labour cost]
YRUT	= YRT(.) / YR(.)	[unit tax cost]
YRUM	= (BQRY(.)*YR(.))* PQRD(.)	[unit material cost]
<i>Restrictions:</i>		
BPYH(.,2 .,3 .,8 .,9) >= 0		['right sign']
BPYH(.,8) + BPYH(.,9) = 1		[long-run cost pass-through]
0 > BPYH(.,6) > -1		['right sign']
<i>Definitions:</i>		
BPYH	is a matrix of parameters	
PQRM	is a matrix of import prices by industry, m Rs at 2010 prices	
YRKE	is a matrix of technological progress by industry	
YRLC	is a matrix of labour costs by industry, m Rs at current prices	
YRT	is a matrix of net taxes by industry, m Rs at current prices	
YR	is a matrix of gross industry output by industry, m Rs at 2010 prices	
QR	is a matrix of gross output by product, m Rs at 2010 prices	
QRX	is a matrix of exports by industry, m Rs at 2010 prices	
BQRY	is a matrix of input-output relationships	
PQRD	is a matrix of prices of sales to domestic markets, 2010 = 1.0	
V-	indicates a current price version of the variable	

## 1.11 Industrial employment

In the econometric representation in E3-India, employment is determined as a function of real output and real wage costs. This is shown in Table 1.21.

The chosen model follows the work of Lee, Pesaran and Pierse (1990) but also incorporates insights from the work on growth theory developed by Scott (1989). A detailed methodological description with empirical results is contained in E3ME working papers no. 28 (Gardiner, 1994) and no. 43 (Barker and Gardiner, 1994). This includes a formal representation of the theoretical optimisation problem for firms to minimise costs for a given level of output.

**Table 1.21: Employment equations**

<i>Co-integrating long-term equation:</i>	
LN(YRE(.))	[total employment]
= BYRE(.,6)	
+ BYRE(.,7) * LN(YR(.))	[real output]
+ BYRE(.,8) * LN(LYLC(.))	[real wage costs]
+ ECM	[error]
<i>Dynamic equation:</i>	
DLN(YRE(.))	[change in total employment]
= BYRE(.,1)	
+ BYRE(.,2) * DLN(YR(.))	[real output]
+ BYRE(.,3) * DLN(LYLC(.))	[real wage costs]
+ BYRE(.,4) * DLN(YRE)(-1)	[lagged change in employment]
+ BYRE(.,5) * ECM(-1)	[lagged error correction]
<i>Identity:</i>	
LYLC = (YRLC(.)/PYR(.)) / YREE(.)	[real labour costs]
<i>Restrictions:</i>	
BYRE(.,2 ..7) >= 0	['right sign']
BYRE(.,3 ..8) <= 0	['right sign']
0 > BYRE(.,5) > -1	['right sign']
<i>Definitions:</i>	
BYRE	is a matrix of parameters
YRE	is a matrix of total employment by industry, in thousands of persons
YR	is a matrix of gross output by industry, m Rs at 2010 prices
YRLC	is a matrix of employer labour costs (wages plus imputed social security contributions) by industry, Rs at current prices
PYR	is a matrix of output prices by industry, 2010=1.0
YREE	is a matrix of wage and salary earners, in thousands of persons

## 1.12 Industrial average earnings

The specification is given in Table 1.22.

The starting point for the equation formation of wage rates used in E3-India is the approach adopted by Lee and Pesaran (1993), which is general enough to accommodate differing degrees of market power on both sides of the labour market. More information is provided in Barker and Gardiner (1996).

The treatment of wage determination is based on a theory of the wage-setting decisions made by a utility-maximising union, where the union derives utility (as the representative of its members) from higher real consumption wages (relative to the fallback level and from higher levels of employment (again relative to a fallback level, which is taken to be proportional to a simple average of employment levels in the last two years in the empirical work). The wage rate is set by unions choosing wage rates to maximise utility subject to the labour-demand constraint imposed by profit-maximising firms. The form of the equation is relatively straightforward: real wages in a sector rise, with weights, if there are internal, sector-specific shocks which cause revenue per worker to rise (e.g. productivity innovations in the sector), or if employment levels are rising; and real wages are also influenced by external effects, including changes in the real wage that can be obtained in the remainder of the economy, changes in incomes received if unemployed, and changes in the unemployment rate itself.

The empirical evidence on the wage equation (surveyed by Layard, Nickell and Jackman, 1991) strongly suggests that, in the long-term, bargaining takes place over real pay, and this is imposed in all the equations presented below. However, in the dynamic equation for the change in wage rates, a response of real rates is allowed and tested by introducing the change in consumer prices. In addition, it has been assumed that long-run price homogeneity holds, so that the long-run economy-wide real product wage rates grow at the same rate as economy-wide labour productivity.

The specification allows for external industry effects on an industry's wage rates, effects of inflation and general economy-wide effects of the unemployment. The parameter on the price index is imposed at unity in all equations, implying that the explanation given is of the real consumer wage.

Table 1.22: The Industrial Average Earnings Equations

<i>Co-integrating long-term equation:</i>	
LN(YRW(.))	[gross nominal average earnings]
= BYRW(.,8)	
+ BYRW(.,9) * LN(YRWE(.))	[external industry wage rates]
+ BYRW(.,10) * LN(PRSC(.))	[consumer price deflator]
+ BYRW(.,11) * DLN(LYRP(.))	[productivity]
+ BYRW(.,12) * LN(RUNR(.))	[unemployment rate]
+ ECM	[error]
<i>Dynamic equation:</i>	
DLN(YRW(.))	[change in gross earnings]
= BYRW(.,1)	
+ BYRW(.,2) * DLN(LYRWE(.))	[external industry wage rates]
+ BYRW(.,3) * DLN(LYRP(.))	[productivity]
+ BYRW(.,4) * DLN(PRSC(.))	[consumer price deflator]
+ BYRW(.,5) * DLN(RUNR(.))	[unemployment rate]
+ BYRW(.,6) * DLN(YRW)(-1)	[lagged change in wage rates]
+ BYRW(.,7) * ECM(-1)	[lagged error correction]
<i>Identities:</i>	
YRWE(.) = SUM OVER I, J (I, J = all other industries and regions)	[external industry wage rates]
(LN(YRW(I)) * YRLC(I) / SUM(YRLC(I)))	
<i>Restrictions:</i>	
BYRW(.,10) = 1	[long-run in real terms]
BYRW(.,2 .,3 .,4 .,9 .,11) >= 0	['right sign']
BYRW(.,5 .,12) <= 0	['right sign']
0 > BYRW(.,7) > -1	['right sign']
<i>Definitions:</i>	
BYRW	is a matrix of parameters
YRW	is a matrix of nominal average earnings (contractual wage) by industry, Rs per person-year
YRLC	is a matrix of nominal employer costs (wages and salaries plus employers' and imputed social security contributions) by industry, Rs at current prices
LYRP	Is a matrix of labour productivity (output per worker)
PRSC	is a vector of the consumer price deflator, 2005 = 1.0
RUNR	is a vector of the standardised unemployment rate

### 1.13 Labour participation rate

The theoretical model for labour force participation rates (see Table 1.23) stems from a paper by Briscoe and Wilson (1992). The standard analysis of participation in the labour force is based around the idea of a reservation wage, such that if the market wage is greater than an individual's reservation wage, they will actively seek employment, and vice versa. It should be noted here that this type of model assumes an excess demand for labour.

Specifically, labour participation rates in E3-India are modelled as a positive function of industry output and average wages. Moreover, they are also negatively related to the evolution of unemployment.

**Table 1.23: The Participation Rate Equations**

Co-integrating long-term equation:	
$\text{LN}(\text{LRP}/(1-\text{LRP}))$	[participation rate, logistic form]
= BLRP(.,6)	
+ BLRP(.,7) * LN(RSQ(.,))	[industry output]
+ BLRP(.,8) * LN(RWS(.,)/(REMP(.,)))	[average wages]
+ BLRP(.,9) * LN(RUNR(.,))	[unemployment rate]
+ ECM	[error]
Dynamic equation:	
$\text{DLN}(\text{LRP}/(1-\text{LRP}))$	[participation rate, logistic form]
= BLRP(.,1)	
+ BLRP(.,2) * DLN(RSQ(.,))	[industry output]
+ BLRP(.,3) * DLN(RWS(.,)/(REMP(.,)))	[average wages]
+ BLRP(.,4) * DLN(RUNR(.,))	[unemployment rate]
+ BLRP(.,5) * ECM(-1)	[lagged error correction]
Identities:	
$\text{LRP} = \text{LABF} / \text{POP}$	[participation rate]
Restrictions:	
$\text{BLRP}(.,2 \dots 3 \dots 7 \dots 8) \geq 0$	[‘right sign’]
$\text{BLRP}(.,4 \dots 9) \leq 0$	[‘right sign’]
$0 > \text{BLRP}(.,5) > -1$	[‘right sign’]
Definitions:	
BLRP	is a matrix of parameters
LRP	is a vector of labour force participation rates by gender and age group
LABF	is a matrix of labour force by gender, in thousands of persons
POP	is a matrix of population of working age by gender, in thousands of persons
RSQ	is a vector of total gross industry output, m Rs at 2010 prices
RWS	is a vector of total wages, m Rs at current prices
RUNR	is a vector of the standardised unemployment rate
REMP	is a vector of total employment, in thousands of persons